# PARTICULAR RELIABILITY CHARACTERISTICS OF TWO-ELEMENT PARALLEL TECHNICAL (MECHATRONIC) SYSTEMS

Zbigniew MATUSZAK Maritime University of Szczecin

#### Abstract:

The paper characterizes the basic distributions of failure of elements that constitute the technical (mechatronical) systems: exponential, Weibull, normal, log-normal distribution. The description of two-element parallel technical systems with reliability characteristics has been made. Specific cases are studied where up time of the technical system elements have exponential, Weibull, normal, log-normal distributions and where the system consists of two parts with parallel reliability structure and of different types of distributions of elements up times. The order of elements in the analysis does not matter. The relevant characteristics of reliability for a system with two parallel elements are presented: up time distribution function of the system, system reliability, up time probability density of the system, the system failure intensity.

**Key words:** technical (mechatronical) system, distributions of elements failure, distributions of failures of two-element parallel technical systems

#### **INTRODUCTION**

The combination of mechanical, electrical, electronic, pneumatic elements into one operating technical system has been called in recent years a mechatronic device (system). Each of these elements has a specific character of durability, damage susceptibility and reliability. Reliability of devices and mechatronic device components is described by mathematical models – distributions of random variables, and particularly, the characteristics of reliability, which directly affect the reliability characteristics of mechatronic devices, into which they are included.

The most commonly used mathematical models of studying the reliability of technical devices are distributions of random variables: exponential, Weibull, normal, logarithmo-normal, normal truncated at zero, gamma, binomial (Bernoulli), Poisson, hypergeometric, geometric, and processes: Poisson, normal, Markov and semi-Markov. Distributions are probabilistic models, and processes are stochastic models [1, 2, 3, 4, 5].

In complex, responsible technical and mechatronic systems (a combination of mechanical, electrical, electronic, pneumatic operating components into a coherent technical system) reserving components are often used. The simplest reserving consists in a parallel inclusion of the same component into the system, which replaces the damaged component

nent at the time of the primary failure. In more complex technical systems, the functions of the defective part may be taken by a different system component, with different operating characteristics, and thus with different reliability characteristics. Each of the components may have a different durability, damage and reliability character, which is described by the mathematical models - statistical distributions, and particularly the characteristics of reliability, which directly affect the reliability characteristics of a technical system they constitute.

The most commonly used mathematical models of studying the reliability of technical devices are distributions of random variables: exponential, Weibull, normal, logarithmo-normal, normal truncated at zero, gamma, binomial (Bernoulli), Poisson, hypergeometric, geometric, and processes: Poisson, normal, Markov and semi-Markov. Distributions are probabilistic models, and processes are stochastic models [1, 2, 3, 4, 5]. In further analysis the simplest and the most common models are assumed: exponential, Weibull, normal, or logarytmo-normal.

Many years of field tests and databases available on failure of components and technical equipment indicate that specific distributions for their reliability characteristics can be attributed to specific components and devices as well as to the typical types of damage [6, 7, 8, 9] (Table 1).

Tabl	le 1.
Functions of failure intensity of selected components and devices as well as typical types of dam	age

Component, device	Distribution of functions of failure intensity	Type of damage	Distribution of functions of failure intensity
small rubber parts such as seals, diaphragms	Weibull	catastrophic	exponential
components and equipment damaged by external factors	exponential	ageing	Weibull, gamma
electronic elements	exponential	very slow wear	exponential
devices with a dominant number of moving parts	Weibull	rapid wear	normal, logarithmo-normal
		corrosive wear	gamma

Up time of the i-th element is a random variable with a distribution defined by the following characteristics:

reliability of the component

$$R_i(t) = P\{\tau_i \ge t\} = 1 - F_i(t), t \ge 0, i = 1, 2, ..., n,$$
(1)

probability density of the component up time

$$f_i(t) = \frac{dF_i(t)}{dt}, t \ge 0, i = 1, 2, ..., n,$$
 (2)

- failure intensity of the component

$$\lambda_{i}(t) = -\frac{d}{dt} \left[ \ln R_{i}(t) \right] = \frac{f_{i}(t)}{1 - F_{i}(t)} = \frac{f_{i}(t)}{R_{i}(t)}, t \ge 0, i = 1, 2, ..., n, \quad (3)$$

- the expected up time of the component

$$T_{i} = E[\tau_{i}] = \int_{0}^{\infty} R_{i}(t) dt, i = 1, 2, ..., n.$$
(4)

In the next part the analysis of four distributions is given as an example, and then selected characteristics of reliability compositions of elements of mechatronic devices are presented. In the given examples, the presentation is limited to compositions of distributions for two elements.

# CHARACTERISTICS OF THE ANALYZED RELIABILITY DISTRIBUTIONS

In the work below [10], particular cases of distribution compositions are analysed.

# Exponential distribution

Exponential distribution is useful for testing the reliability of such devices, which are the result of impact of shock loads (so-called discrete stimuli). Exponential distribution can be used to test the reliability of equipment and components if:

- changes to the technical condition and the resulting damage is irreversible,
- the level of resistance (wear resistance) is constant, which means no damage caused by aging (derived from cumulative extortion),
- damage is the result of external or internal random shock interactions (discrete stimuli).

Up time characteristics (i=1,2,...,n) of a component are as follows:

- reliability of the component

$$R_i(t) = e^{-\lambda_i t}, t \ge 0, \tag{5}$$

probability density of the component up time

$$f_i(t) = \lambda_i e^{-\lambda_i t}, t \ge 0, \tag{6}$$

failure intensity of the component

$$\lambda_i(t) = \lambda_i = const. \tag{7}$$

the expected up time of the component

$$T_i = E\left[\tau_i\right] = \frac{1}{\lambda_i}.\tag{8}$$

#### Weibull distribution

Weibull distribution describes the time of normal operation of such devices, in which damage is independent, any damage causes loss of equipment up time, each unit consists of a sufficiently large number of homogeneous components.

The element has a Weibull distribution with the parameters  $(\alpha_i,\beta_i)$ (i=1,2,...,n), when its characteristics take the form:

reliability of the component

$$R_i(t) = exp(-\beta_i t^{\alpha_i}) t \ge 0, \tag{9}$$

- probability density of the component up time

$$f_i(t) = \alpha_i \beta_i t^{\alpha_i - 1} \exp(-\beta_i t^{\alpha_i}), t \ge 0, \tag{10}$$

- failure intensity of the component

$$\lambda_i(t) = \alpha_i \beta_i t^{\alpha_i - 1}, t \ge 0, \tag{11}$$

the expected up time of the component

$$T_{i} = E\left[\tau_{i}\right] = \Gamma\left(1 + \frac{1}{\alpha_{i}}\right)\beta_{i}^{-\frac{1}{\alpha_{i}}}, t \geq 0.$$
 (12)

A special case of Weibull distribution is a Rayleigh distribution in which the parameter  $\alpha_i$ =2.

# Normal distribution

The normal distribution is a model of reliability of any technical object in which there are damages resulting from the aging process, including wear. This is to be used if a described random variable depends on a number of phenomena and causes, none of which can be considered dominant.

The component  $\tau_i$  up time has a normal distribution when the probability density has the form

$$f_i(t) = \frac{1}{\sigma_i \sqrt{2\pi}} exp \left[ -\frac{(t - T_i)^2}{2\sigma_i^2} \right], -\infty < t < \infty,$$
 (13)

and its distribution function

$$F_i(t) = \frac{1}{\sigma_i \sqrt{2\pi}} \int_{-\infty}^{t} exp \left[ -\frac{(x - T_i)^2}{2\sigma_i^2} \right] dx, \quad \infty < t < \infty, \quad (14)$$

where  $T_i$  is the expected up time of a component and  $\sigma_i^2$  is its variance.

The normal distribution is defined for all  $t \in R$ , while the random variable  $\tau_i$  being the up time of the element takes only nonnegative values. One can put up with such an inadequacy of the model, where the probabilities  $P\{\tau_i < 0\}$  are negligibly small, no larger than measurement errors.

In order to write simpler and more convenient up time characteristics of the element with normal distribution the characteristics of the normal distribution N (0,1) probability density has been used:

$$f_0(t) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{t^2}{2}\right), t \in R,$$
 (15)

and the distribution function

$$F_0(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} exp\left(-\frac{x^2}{2}\right) dx, t \in R.$$
 (16)

One can then write the characteristics of the system components in the form:

probability density of the component up time

$$f_i(t) = \frac{1}{\sigma_i} f_0 \left( \frac{t - T_i}{\sigma_i} \right), t \in R, \tag{17}$$

reliability of the component

$$R_i(t) = 1 - F_0 \left(\frac{t - T_i}{\sigma_i}\right) \tag{18}$$

failure intensity of the component

$$\lambda_{i}(t) = \frac{f_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)}{\sigma_{i}\left[1 - F_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)\right]}$$
(19)

In case one cannot accept the assumption that the probabilities  $P\{\tau_i < 0\}$  are negligibly small, one should use the truncated normal distribution, at which time the fault-free operation of the component takes only nonnegative values  $\tau_i \ge 0$ 

In this case, the reliability characteristics of elements has the form:

probability density of the component uptime

$$f_{i}(t) = \frac{\frac{1}{\sigma_{i}} f_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)}{1 - F_{0}\left(-\frac{T_{i}}{\sigma_{i}}\right)} = \frac{f_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)}{\sigma_{i} F_{0}\left(\frac{T_{i}}{\sigma_{i}}\right)}, t \ge 0, \tag{20}$$

reliability of the component

$$R_{i}(t) = 1 - \frac{F_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right) - F_{0}\left(-\frac{T_{i}}{\sigma_{i}}\right)}{1 - F_{0}\left(-\frac{T_{i}}{\sigma_{i}}\right)} = \frac{1 - F_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)}{F_{0}\left(\frac{T_{i}}{\sigma_{i}}\right)} = \frac{F_{0}\left(\frac{T_{i} - t}{\sigma_{i}}\right)}{F_{0}\left(\frac{T_{i}}{\sigma_{i}}\right)} \tag{21}$$

failure intensity of the component

$$\lambda_{i}(t) = \frac{f_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)}{\sigma_{i}\left[1 - F_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)\right]} = \frac{f_{0}\left(\frac{t - T_{i}}{\sigma_{i}}\right)}{\sigma_{i}F_{0}\left(\frac{T_{i} - t}{\sigma_{i}}\right)}, t \ge 0. \quad (22)$$

Logarithmo-normal distribution

Logarithmo-normal distribution in reliability theory on the basis of the empirical research characterizes metal components safe fatigue life, the strength of metals subjected to prolonged operation stress, as well as electronic components up time.

The component  $\tau_i$  up time has logarithmo-normal distribution when the random variable  $Y=ln\tau_i$  is normally distributed withthe parameters  $N(T_i,\sigma_i)$ . Using the probability density and distribution function of the normal distribution N(0,1) component reliability characteristics of logarithmonormal distribution can be written as:

probability density of the component up time

$$f_i(t) = \frac{1}{t\sigma_i\sqrt{2\pi}}exp\left[-\frac{\left(\ln t - T_i\right)^2}{2\sigma_i^2}\right] = \frac{1}{t\sigma_i}f_0\left(\frac{\ln t - T_i}{\sigma_i}\right), t > 0, \quad (23)$$

- reliability of the component

$$R_{i}(t) = 1 - F_{0}\left(\frac{lnt - T_{i}}{\sigma_{i}}\right) = F_{0}\left(\frac{T_{i} - lnt}{\sigma_{i}}\right), \tag{24}$$

failure intensity of the component

$$\lambda_i(t) = \frac{\frac{1}{t\sigma_i^2} f_0\left(\frac{\ln t - T_i}{\sigma_i}\right)}{F_0\left(\frac{T_i - \ln t}{\sigma_i}\right)},\tag{25}$$

the expected up time of the component

$$E\left[\tau_{i}\right] = exp\left(T_{i} + \frac{\sigma_{i}^{2}}{2}\right) \tag{26}$$

# CHARACTERISTICS OF THE DISTRIBUTIONS OF TWO-ELEMENT PARALLEL TECHNICAL SYSTEMS

Here the analysis of n-element system with a parallel reliability structure is presented. The same designations, which have been defined in the previous section, will be used. Since the elements are independent, and the system

is working flawlessly until the damage of all the components:

the up time distribution is

$$F(t) = P\{\tau < t\} = P\{\tau_1 < t, \tau_2 < t, ..., \tau_n < t\} = \prod_{i=1}^{n} F_i(t),$$
 (27)

the system reliability

$$R(t) = 1 - \prod_{i=1}^{n} F_i(t),$$
 (28)

probability density of the system up time

$$f(t) = \sum_{i=1}^{n} f_i(t) \prod_{\substack{j=1 \ j \neq i}} F_j(t),$$
 (29)

Failure intensity of the system has the form

$$\lambda(t) = \frac{\sum_{i=1}^{n} f_i(t) \prod_{\substack{j=1\\j \neq i}} F_j(t)}{1 - \prod_{i=1}^{n} F_i(t)}$$
(30)

The expected up time of the system with parallel reliability structure is

$$T = E[\tau] = \int_{0}^{\infty} \left[ 1 - \prod_{i=1}^{n} F_i(t) \right] dt$$
 (31)

The equation (31) shows that it is rarely possible to calculate the expected up time of the system in the explicit form. Even for simple distributions the expected up time of the system is quite complicated. In a particular case, all components of the system have the same up time distributions. This happens usually when several components perform one and the same function.

To perform it one component is sufficient, therefore, the remaining elements are hot reserving and in that case often  $F_i(t)=F_1(t)$  for i=1,2,...,n. Then for a system with a parallel structure consisting of identical components reliability characteristics have the form:

the distribution function of the system up time

$$F(t) = F_1^n(t), \tag{32}$$

the system reliability

$$R(t) = 1 - F_1^n(t) = 1 - \left[1 - R_1(t)\right]^n, \tag{33}$$

probability density of the system up time

$$f(t) = nf_1(t)F_1^{n-1}(t),$$
 (34)

failure intensity of the system

$$\lambda(t) = \frac{nf_1(t)F_1^{n-1}(t)}{1 - F_1^{n}(t)},\tag{35}$$

the expected up time of the system

$$T = E[\tau] = \int_{0}^{\infty} [1 - F_{1}^{n}(t)] dt$$
 (36)

The following reliability characteristics of systems with a parallel two-element structure and different types of components up time distributions have been determined. The order of components is arbitrary, it does not matter for the dependence.

The first component has the up time of the exponential distribution with the parameter  $\lambda$ , and the second – of the Weibull distribution with the parameters  $(\alpha,\beta)$ . Reliability characteristics of the system take the form:

the distribution function of the system up time

$$F(t) = (1 - e^{-\lambda t})(1 - e^{-\beta t^{\alpha}})t \ge 0,$$
 (37)

- the system reliability

$$R(t) = 1 - \left(1 - e^{-\lambda t}\right)\left(1 - e^{-\beta t^{\alpha}}\right) = e^{-\lambda t} + e^{-\beta t^{\alpha}} - e^{-(\lambda t + \beta t^{\alpha})}, (38)$$

probability density of the system up time

$$f(t) = \lambda e^{-\lambda t} \left( 1 - e^{-\beta t^{\alpha}} \right) + \alpha \beta t^{\alpha - 1} \left( 1 - e^{-\lambda t} \right) e^{-\beta t^{\alpha}} =$$

$$= \lambda e^{-\lambda t} + \alpha \beta t^{\alpha - 1} e^{-\beta t^{\alpha}} - \left( \lambda + \alpha \beta t^{\alpha - 1} \right) e^{-(\lambda t + \beta t^{\alpha})},$$
(39)

failure intensity of the system

$$\lambda(t) = \frac{\lambda e^{-\lambda t} + \alpha \beta t^{\alpha - 1} e^{-\beta t^{\alpha}} - \left(\lambda + \alpha \beta t^{\alpha - 1}\right) e^{-(\lambda t + \beta t^{\alpha})}}{e^{-\lambda t} + e^{-\beta t^{\alpha}} - e^{-(\lambda t + \beta t^{\alpha})}}, \quad (40)$$

The first component has the up time of the exponential distribution with the parameter  $\lambda$ , and the second – of the Weibull distribution with the parameters  $(T,\sigma)$ . Reliability characteristics of the system take the form:

the distribution function of the system up time

$$F(t) = \left(1 - e^{-\lambda t}\right) F_0\left(\frac{t - T}{\sigma}\right) \tag{41}$$

the system reliability

$$R(t) = 1 - \left(1 - e^{-\lambda t}\right) F_0(\frac{t - T}{\sigma}),$$
 (42)

- probability density of the system up time

$$f(t) = \lambda e^{-\lambda t} F_0(\frac{t-T}{\sigma}) + \frac{1}{\sigma} \left( 1 - e^{-\lambda t} \right) f_0(\frac{t-T}{\sigma}), \tag{43}$$

- failure intensity of the system

$$\lambda(t) = \frac{\lambda e^{-\lambda t} F_0(\frac{t-T}{\sigma}) + \frac{1}{\sigma} \left(1 - e^{-\lambda t}\right) f_0(\frac{t-T}{\sigma})}{1 - \left(1 - e^{-\lambda t}\right) F_0(\frac{t-T}{\sigma})} \tag{44}$$

The first component has the up time of the exponential distribution with the parameter  $\lambda$ , and the second – of the logarithmo-normal distribution with the parameters  $(T,\sigma)$ . Reliability characteristics of the system take the form:

the distribution function of the system up time

$$F(t) = \left(1 - e^{-\lambda t}\right) F_0\left(\frac{\ln t - T}{\sigma}\right), t \ge 0, \tag{45}$$

the system reliability

$$R(t) = 1 - (1 - e^{-\lambda t}) F_0(\frac{\ln t - T}{\sigma}),$$
 (46)

- probability density of the system up time

$$f(t) = \lambda e^{-\lambda t} F_0\left(\frac{\ln t - T}{\sigma}\right) + \frac{1}{t\sigma} \left(1 - e^{-\lambda t}\right) f_0\left(\frac{\ln t - T}{\sigma}\right),\tag{47}$$

failure intensity of the system

$$\lambda(t) = \frac{\lambda e^{-\lambda t} F_0(\frac{\ln t - T}{\sigma}) + \frac{1}{t\sigma} \left(1 - e^{-\lambda t}\right) f_0(\frac{\ln t - T}{\sigma})}{1 - \left(1 - e^{-\lambda t}\right) F_0(\frac{\ln t - T}{\sigma})} \tag{48}$$

The first component has the up time of the Weibull distribution with the parameter  $(\alpha,\beta)$ , and the second – of the normal distribution with the parameters  $(T,\sigma)$ . Reliability characteristics of the system take the form:

- the distribution function of the system up time

$$F(t) = \left(1 - e^{-\beta t^{\alpha}}\right) F_0\left(\frac{t - T}{\sigma}\right), t \ge 0, \tag{49}$$

the system reliability

$$R(t) = 1 - \left(1 - e^{-\beta t^{\alpha}}\right) F_0\left(\frac{t-T}{\sigma}\right), \tag{50}$$

- probability density of the system up time

$$f(t) = \alpha \beta t^{\alpha - 1} e^{-\beta t^{\alpha}} F_0(\frac{t - T}{\sigma}) + \frac{1}{\sigma} \left( 1 - e^{-\beta t^{\alpha}} \right) f_0(\frac{t - T}{\sigma}), \tag{51}$$

- failure intensity of the system

$$\lambda(t) = \frac{\alpha \beta t^{\alpha - 1} e^{-\beta t^{\alpha}} F_0(\frac{t - T}{\sigma}) + \frac{1}{\sigma} \left( 1 - e^{-\beta t^{\alpha}} \right) f_0(\frac{t - T}{\sigma})}{1 - \left( 1 - e^{-\beta t^{\alpha}} \right) F_0(\frac{t - T}{\sigma})} \tag{52}$$

The first component has the up time of the Weibull distribution with the parameter  $(\alpha,\beta)$ , and the second – of the logarithmo-normal distribution with the parameters  $(T,\sigma)$ . Reliability characteristics of the system take the form:

- the distribution function of the system up time

$$F(t) = \left(1 - e^{-\beta t^{\alpha}}\right) F_0\left(\frac{\ln t - T}{\sigma}\right), t \ge 0, \tag{53}$$

- the system reliability

$$R(t) = 1 - \left(1 - e^{-\beta t^{\alpha}}\right) F_0\left(\frac{\ln t - T}{\sigma}\right), t \ge 0,$$
 (54)

- probability density of the system up time

$$f(t) = \alpha \beta t^{\alpha - 1} e^{-\beta t^{\alpha}} F_0(\frac{\ln t - T}{\sigma}) + \frac{1}{t\sigma} \left( 1 - e^{-\beta t^{\alpha}} \right) f_0(\frac{\ln t - T}{\sigma}), \tag{55}$$

- failure intensity of the system

$$\lambda(t) = \frac{\alpha \beta t^{\alpha - 1} e^{-\beta t^{\alpha}} F_0(\frac{\ln t - T}{\sigma}) + \frac{1}{t\sigma} \left(1 - e^{-\beta t^{\alpha}}\right) f_0(\frac{\ln t - T}{\sigma})}{1 - \left(1 - e^{-\beta t^{\alpha}}\right) F_0(\frac{\ln t - T}{\sigma})}$$
(56)

The first component has the up time of the normal distribution with the parameter  $(T_1,\sigma_1)$ , and the second – of the logarithmo-normal distribution with the parameters  $(T_2,\sigma_2)$ . Reliability characteristics of the system take the form:

- the distribution function of the system up time

$$F(t) = F_0 \left(\frac{t - T_1}{\sigma_1}\right) F_0 \left(\frac{\ln t - T_2}{\sigma_2}\right), t \ge 0, \tag{57}$$

- the system reliability

$$R(t) = 1 - F_0 \left(\frac{t - T_1}{\sigma_1}\right) F_0 \left(\frac{\ln t - T_2}{\sigma_2}\right), t \ge 0,$$
 (58)

probability density of the system up time

$$f(t) = \frac{1}{\sigma_1} f_0 \left( \frac{t - T_1}{\sigma_1} \right) F_0 \left( \frac{\ln t - T_2}{\sigma_2} \right) + \frac{1}{t \sigma_2} F_0 \left( \frac{t - T_1}{\sigma_1} \right) f_0 \left( \frac{\ln t - T_2}{\sigma_2} \right)$$
 (59)

failure intensity of the system

$$\lambda(t) = \frac{\frac{1}{\sigma_1} f_0(\frac{t-T_1}{\sigma_1}) F_0(\frac{\ln t - T_2}{\sigma_2}) + \frac{1}{t\sigma_2} F_0(\frac{t-T_1}{\sigma_1}) f_0(\frac{\ln t - T_2}{\sigma_2})}{1 - F_0(\frac{t-T_1}{\sigma_1}) F_0(\frac{\ln t - T_2}{\sigma_2})}$$
(60)

#### **CONCLUDING REMARKS**

In technical systems, particularly in mechatronic systems, reserving the system components is popular. In the presented material specific examples of components reserving are shown, when the components are not reserved by the objects of the same type, with the same characteristics of reliability, but when the functional reserving by a different component with a different reliability characteristics is used. In case of reserving by the same components with the same failure distributions, the analysis of reliability characteristics is relatively simple and defined by the dependences (32),(35). In other cases it is complex.

### **REFERENCES**

- [1] Prażewska M. (ed.): Niezawodność urządzeń elektronicznych. WKiŁ. Warszawa 1987.
- [2] Rausand M., Høyland A.: System Reliability Theory: Models, Statistical Methods, and Applications. Second edition. Interscience. New Jersey: Wiley 2004.
- [3] Saleh J. H., Marais K.: Reliability: How much is it worth? Beyond its estimation or prediction, the (net) present value of reliability. Reliability Engineering and System Safety. Vol. 91, 2006, pp. 665–673.
- [4] Sotskow B.S.: Niezawodność elementów i urządzeń automatyki. WNT. Warszawa 1973.
- [5] Ważyńska-Fiok K., Jaźwiński J.: Niezawodność systemów technicznych. PWN. Warszawa 1990.
- [6] Matuszak Z.: Charakterystyki niezawodnościowe wieloelementowych struktur mieszanych i odnawialnych. Collection of research papers of the Baltic Association of Mechanical Engineering Experts No 4, Mechanical Engineering of the Baltic Region. Kaliningrad State Technical University. Kaliningrad 2004, pp. 146-148.
- 7] Matuszak Z.: Charakterystyki niezawodnościowe kilkuelementowych systemów technicznych o strukturze szeregowej i równoległej o różnych rozkładach czasów zdatności. Collection of research papers of the Baltic Association of Mechanical Engineering Experts No 4, Mechanical Engineering of the Baltic Region. Kaliningrad State Technical University. Kaliningrad 2004, pp. 149-153.

- [8] Matuszak Z.: Selected safety models of elements and systems in the engine room. International Scientific Journal "Problems Of Applied Mechanics". Georgian Committee of International Federation for the Machines and Mechanics. Tbilisi 2004, No 1(14), 2004, pp. 30-39.
- [9] Nowakowski T.: Bazy wiedzy w badaniach niezawodności maszyn. Materiały Konferencji Naukowej
- dr hab. inż. Zbigniew Matuszak, prof AM Maritime University of Szczecin Faculty of Mechanical Engineering Institute of Ship Power Plant Operation ul. Wały Chrobrego 1-2, 70-500 Szczecin, POLAND

e-mail: z.matuszak@am.szczecin.pl

- "Metody doświadczalne w budowie i eksploatacji maszyn roboczych, technologicznych oraz środków transportu". Wydawnictwo Politechniki Wrocławskiej. Wrocław-Szklarska Poręba 1993, s. 219-225.
- [10] Matuszak Z.: Szczególne charakterystyki niezawodnościowe szeregowych systemów mechatronicznych. Studia i Materiały Polskiego Stowarzyszenia Zarządzania Wiedzą. Nr 45, 2011, s. 176-190.